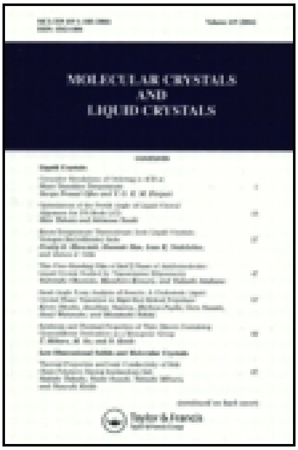

Molecular Crystals and Liquid Crystals
Publication details, including instructions for authors and subscription information:
http://www.tandfonline.com/loi/gmcl20

Distribution of Diffraction Intensity in Diffraction Orders for Thin Anisotropic Diffraction Gratings

Gegham G. Zakharyan [a] , Rafik S. Hakobyan [a] & Artur V. Galstyan [a]
[a] Department of Physics, Yerevan State University, Yerevan, Armenia

Version of record first published: 22 Sep 2010

Mol. Cryst. Liq. Cryst., Vol. 488, pp. 110–117, 2008

ISSN: 1542-1406 print/1563-5287 online
DOI: 10.1080/15421400802240409

Taylor & Francis
Taylor & Francis Group

Distribution of Diffraction Intensity in Diffraction Orders for Thin Anisotropic Diffraction Gratings

Gegham G. Zakharyan, Rafik S. Hakobyan, and Artur V. Galstyan
Department of Physics, Yerevan State University, Yerevan, Armenia

The Raman-Nath diffraction in thin anisotropic slanted holographic gratings is studied. Also, dielectric and absorption modulation with common grating vector and of arbitrary relative phase shift is considered. Solutions for the wave amplitudes, diffraction efficiencies are given in transmission geometries for the case of dielectric and absorption modulations. The effect of slanted fringes of diffraction gratings is discussed and for comparison is presented the dependences for diffraction gratings in unslanted cases too. This theoretical method can be extended to a study of the mechanism of distribution and control of diffraction intensities within diffraction orders in materials such as organic crystals, ordered polymers, liquid crystalline and polymer-dispersed liquid crystalline cells.

Keywords: anisotropy; slanted gratings; thin diffraction gratings

1. INTRODUCTION

Recently there has been an increasing interest in the use of holographic diffraction gratings for various optical applications, such as narrowband optical filters, optical data storages, various diffractive elements, laser beam control devices, holographic Stokesmeters, etc. The theoretical efforts to understand light diffraction in holographic media have been investigated by many researchers [1–7]. For thick gratings, light diffraction in isotropic media has developed in the coupled wave theory of Kogelnik [1]. The light diffraction at mixed phase and absorption anisotropic thick media has been given by Montemezzani and Zgonik [2]. Many investigators defined thick and thin gratings regime. However, the more exact definition of a thick and thin grating has been given by Gaylord and Moharam [8]. Many

Address correspondence to Gegham G. Zakharyan, Department of Physics, Yerevan State University, Yerevan 375049, Armenia. E-mail: gegzakh@yahoo.com

approximate methods of solving Maxwell's equations in an inhomogeneous medium to obtain approximate diffraction formulas or relations are known (the WKB approximation, the phase function method), because it is difficult to obtain the correct solution. The theory of light diffraction in acoustically produced thin media (refractive index gratings) has been presented by Raman and Nath [4], which is correctly for isotropic materials. In [9] presented a general formulation of the corrections to the Raman-Nath approximation for the diffracted field by volume holograms, based on integral equations. For anisotropic thin media the theory of light diffraction for unslanted refractive index and absorption gratings we have investigated in our previously works [5–7].

In this work we have studied the light diffraction in mixed phase and absorption thin transmission gratings. We consider the diffraction grating with slanted fringes and relative phase shift between phase and absorption modulations. We discussed the effect of slanted fringes of diffraction grating and have shown its advantage at the help of figures.

2. PRESENTATION OF THE COUPLE–WAVE EQUATIONS

Let's consider a thin anisotropic diffraction grating, which contains the phase and absorption modulations. As well, we will take into consideration the relative phase shift between phase and absorption modulations and therefore, the dielectric tensor can be expressed as

$$\overleftrightarrow{\varepsilon} = \overleftrightarrow{\varepsilon}_r^0 + \overleftrightarrow{\varepsilon}_r^1 \sin(\vec{K}\cdot\vec{r}) + i[\overleftrightarrow{\varepsilon}_i^0 + \overleftrightarrow{\varepsilon}_i^1 \sin(\vec{K}\cdot\vec{r}+\delta)] \tag{1}$$

where the superscripts 0 and 1 denote the constant and the modulated components, respectively, the subscripts r and i denote the real and imaginary parts of dielectric permittivity, δ is a relative phase shift between refractive index modulation and absorption modulation. We consider a case when grating vector $|\vec{K}| = 2\pi/\Lambda$ has arbitrary direction (see Fig. 1), and incident light assumed to be monochromatic and linearly polarized, Λ is a grating period. The total electric field amplitude is given by a sum of plane waves, which are propagating along m-orders [5,6]

$$\vec{E}(\vec{r},t) = \sum_m \vec{E}_m(\vec{r})\exp(i\vec{k}_m\vec{r})\exp(-i\omega t) + c.c. \tag{2}$$

where $\vec{E}_m$ are complex amplitudes of diffracted m-order. In absorption crystals the wave vectors $\vec{k}_m$ are complex with the imaginary part, which possibly has a different direction than the real part [10]

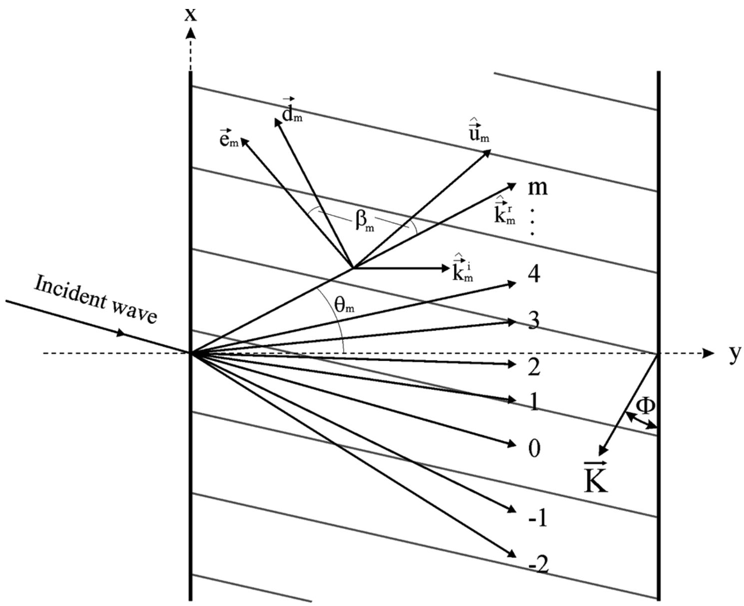

FIGURE 1 Diagram illustrating vectors and angles.

$$\vec{k}_m = \vec{k}^r_m + i\vec{k}^i_m \tag{3}$$

The real part, as usual, is related to the wave-front propagation direction for an eigenpolarization in the crystal, but the imaginary part is related to the absorption experienced by the waves [2]. The total electric field amplitude (2) has to satisfy the time-independent vector wave equation

$$\vec{\nabla} \times (\vec{\nabla} \times \vec{E}) - k_0^2 \overleftrightarrow{\varepsilon}\vec{E} = 0 \tag{4}$$

where $k_0 = \omega/c$ is the free-space wave number. In our case of thin gratings the momentum conservation equation we can presented as follows

$$\vec{k}_m = \vec{k}_i + m\vec{K} \tag{5}$$

where $\vec{k}_i = \vec{k}^r_i + i\vec{k}$ is the complex wave vector of incident wave. Now, let's insert Eqs. (1) and (2) into the wave Eq. (3). Using the above arguments and some vector algebra, Eq. (4) transforms in the following equation

$$\sum_m \Big[-i\Big\{ (\vec{\nabla}\times\vec{E}_m)\times\vec{k}_m + \vec{\nabla}(\vec{E}_m\times\vec{k}_m)\Big\} - E_m\Delta_m\vec{d}_m\sin(\vec{e}_m\vec{k}_m)\Big]\exp(i\vec{k}_m\vec{r})$$
$$= \sum_m \Big\{ k_0^2\overleftrightarrow{\varepsilon}_r^1\vec{E}_m\frac{1}{2i}\exp(i(m+1)\vec{K}\vec{r}) - k_0^2\overleftrightarrow{\varepsilon}_r^1\vec{E}_m\frac{1}{2i}\exp(i(m-1)\vec{K}\vec{r})$$
$$+ ik_0^2\overleftrightarrow{\varepsilon}_i^1\vec{E}_m\frac{1}{2i}\exp(i\delta)\exp(i(m+1)\vec{K}\vec{r}) - ik_0^2\overleftrightarrow{\varepsilon}_i^1\vec{E}_m\frac{1}{2i}$$
$$\times\exp(-i\delta)\exp(i(m-1)\vec{K}\vec{r})\Big\} \tag{6}$$

In this equation, Δ_m is the phase detuning from the Bragg condition for diffracted m-order and presented following way

$$\Delta_m = m^2K^2 + 2m\vec{K}\vec{k}_i = m^2K^2 - 2mKk_i^r\sin(\theta_i+\Phi) \tag{7}$$

where Φ is a slanted angle (angle between grating vector $\vec{K}$ and axis x, see Fig. 1), θ_i is a incident angle inside the grating (internal angle). After mathematical transformations, the final couple-wave equations are given by [5]

$$E'_m(y) + im\rho_m E_m(y) = \frac{\xi_{m-1}}{2}E_{m-1}(y) - \frac{\xi_{m+1}}{2}E_{m+1}(y) \tag{8}$$

where we make following quantities

$$\begin{aligned}\frac{\xi_{m-1}}{2} &\equiv \frac{k_0}{4n_m g_m\cos\varphi_m}\left[A^r_{m-1} + iA^i_{m-1}\exp(i\delta)\right]\\ \frac{\xi_{m+1}}{2} &\equiv \frac{k_0}{4n_m g_m\cos\varphi_m}\left[A^r_{m+1} + iA^i_{m+1}\exp(-i\delta)\right]\end{aligned} \tag{9}$$

and

$$\rho_m \equiv -\frac{g_m(mK^2 + 2\vec{K}\vec{k}_i)}{2k_0n_m\cos\varphi_m} = \frac{g_m(2Kk_i^r\sin\theta_i - mK^2)}{2k_0n_m\cos\varphi_m} \tag{10}$$

which is parameter that describe phase detuning from Bragg condition. In Eq. (9), $A^r_{m\pm1}$ and $A^i_{m\pm1}$ is a parameters that describe phase and absorption modulations, respectively

$$\begin{aligned}A^r_{m\pm1} &= \vec{e}_m\overleftrightarrow{\varepsilon}_r^1\vec{e}_{m\pm1} = \vec{e}_{m\pm1}\overleftrightarrow{\varepsilon}_r^1\vec{e}_m\\ A^i_{m\pm1} &= \vec{e}_m\overleftrightarrow{\varepsilon}_i^1\vec{e}_{m\pm1} = \vec{e}_{m\pm1}\overleftrightarrow{\varepsilon}_i^1\vec{e}_m\end{aligned} \tag{11}$$

In Eqs. (9), (10) and (11) other quantities are following: $\varphi_m = \theta_m + \arccos(g_m)$ is angle between the normal to the surface and Poynting vector for the m-order diffracted beam, θ_m is a diffraction angle of m-order diffracted beam, n_m is a refractive index of m-order diffracted beam, g_m is a projection cosines between $\vec{k}_m$ and Poynting vector for m-order diffracted beam, $\vec{e}_m$ is a unit vector along electric field vector $\vec{E}_m$. For the solution of the Eq. (8) we shall search so [5,6]

$$E_m(y) = E_0 \exp\left(-i\frac{1}{2}m\rho_m y\right)U_m(y) \tag{12}$$

Applying the following approximation $\rho_{m-1} \approx \rho_{m+1} \approx \rho_m$, change the variable in the following way [5]

$$\chi = \frac{1}{i\rho_m}\left\{\xi_{m-1}\exp\left(i\frac{1}{2}\rho_m y\right) - \xi_{m+1}\exp\left(-i\frac{1}{2}\rho_m y\right)\right\}, \tag{13}$$

The final solution of couple-wave equations can be presented in the following form

$$E_m = E_0 \exp\left(-i\frac{1}{2}m\rho_m y\right) \times J_m\left(\frac{1}{i\rho_m}\left\{\xi_{m-1}\exp\left(i\frac{1}{2}\rho_m y\right) - \xi_{m+1}\exp\left(-i\frac{1}{2}\rho_m y\right)\right\}\right) \tag{14}$$

We will define m-order diffraction efficiency as earlier [5]

$$\eta_m = \frac{I_m(y=d)}{I_i(y=0)} = \frac{E_m(y=d)E_m^*(y=d)}{E_i(y=0)E_i^*(y=0)}\frac{n_m g_m \cos\varphi_m}{n_i g_i \cos\varphi_i}\exp(-2k_m^i d) \tag{15}$$

where d is a grating thickness, $\exp(-2k_m^i d)$ multiplying factor is describe the absorption effects during diffraction process.

3. NUMERICAL CALCULATIONS

In numerical calculations we assume that incident light is linearly polarized in incident surface (p-polarization). The grating parameters are chosen so that took place Raman-Nath diffraction regime. Corresponding parameters [8], which are describing diffraction regime are follows: $Q = 0.4 < 1$ and $Q \cdot \gamma = 0.87 \leq 1$. In Figures 2 and 3 show the diffraction efficiencies for 1, 2, 3 diffraction orders depend on incident angle, for two different slanted angles: $\Phi = 10^\circ$ and $\Phi = 20^\circ$. Figure 4 show diffraction efficiency depend on incident angle for $+1$ and -1 diffraction orders, when slanted angle $\Phi = 35^\circ$. This diffraction orders is more interesting, because they are the smooth and intensive beams. We can see, that this orders differ only angular mismatch. Figure 5 show $m = 0$ order diffraction efficiency depend on incident angle, for

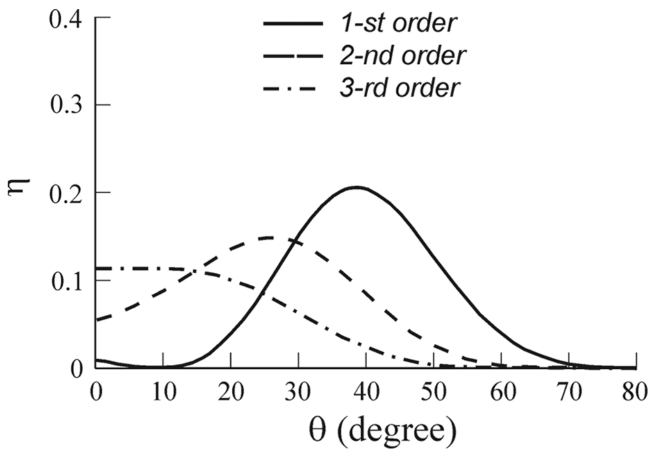

FIGURE 2 Diffraction efficiency vs. incident angle, $\Phi = 10^{\circ}$.

various slanted angles: $\Phi = 0^{\circ}$, $\Phi = 20^{\circ}$, $\Phi = 40^{\circ}$. Particularly, the effect of slanted angle can be presented as follows: when we have unslanted grating, we cannot reproduce the case where angle between wave vector of incident beam and grating fringes will be smaller than 90°, which in some special cases happens very necessary from the point of view of maximums of diffraction efficiency. With the help of slanted fringes we can reproduce the necessary value of this angle

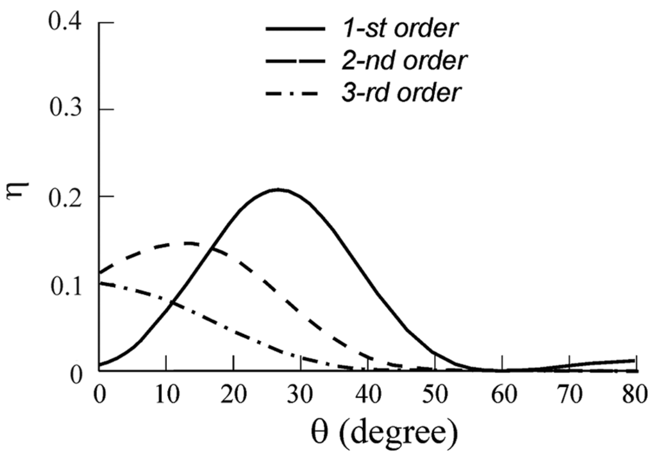

FIGURE 3 Diffraction efficiency vs. incident angle, $\Phi = 20^{\circ}$.

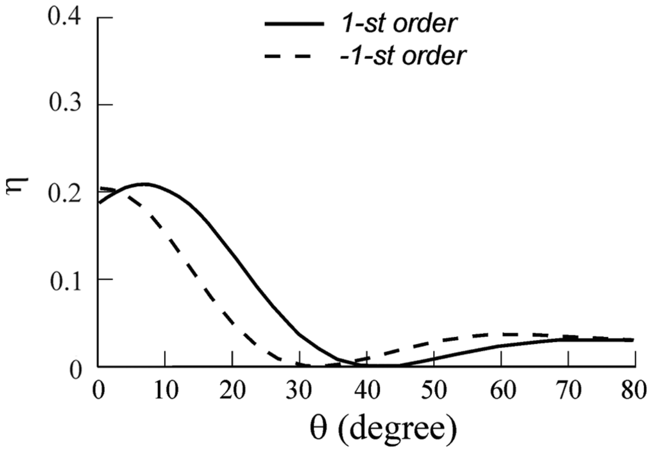

FIGURE 4 Diffraction efficiency vs. incident angle, $\Phi = 35^{\circ}$.

and therefore have the maximum of diffraction efficiency at desirable incident angle.

4. CONCLUSION

We presented a couple-wave theory for thin anisotropic slanted gratings with refractive index and absorption modulations. Dependences

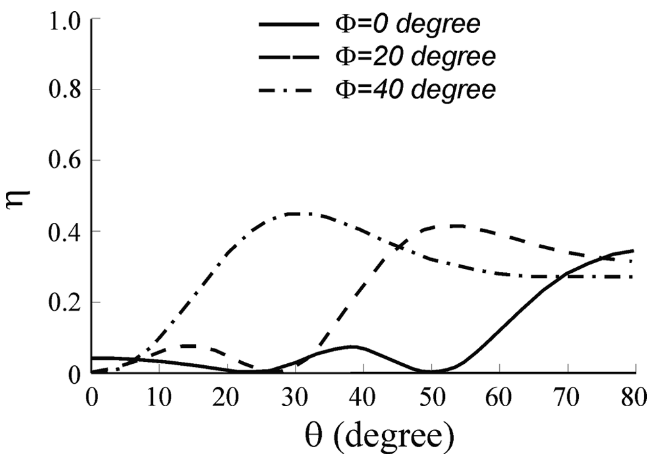

FIGURE 5 Diffraction efficiency vs. incident angle, for $m = 0$ diffraction order.

presented for various diffraction orders, and various slanted angles. We described the effect of slanted diffraction gratings, which is displayed in figures. Also, we presented the unslanted grating cases too.

REFERENCES

[1] Kogelnik, H. (1969). *Bell Syst. Tech. J.*, *48*, 2909.
[2] Montemezzani, G. & Zgonik, M. (1997). *Phys. Rev. E*, *55*, 1035.
[3] Kojima, K. (1982). *Jpn. J. Appl. Phys.*, *21*, 1303.
[4] Raman, C. V. & Nath, N. S. N. Proc. Indian Acad. Sci., Pt. I 2A, 406, (1935), Pt. II 2A, 413 (1935), Pt. III 3A, 75 (1936), Pt. IV 3A, 119 (1936), Pt. V 3A, 459 (1936). Nath, N. S. N. (1937). *Proc. Indian Acad. Sci.*, *4A*, 222.
[5] Galstyan, A. V., Zakharyan, G. G., & Hakobyan, R. S. (2006). *Mol. Cryst. Liq. Cryst.*, *453*, 203.
[6] Zakharyan, G. G. & Galstyan, A. V. (2007). *Opto-Electron. Rev.*, *15*, 20.
[7] Zakharyan, G. G. First European Topical Meeting on Nanophotonics and Metamaterials, NANOMETA 2007, Olympia Congress Centre, Seefeld, Tirol, Austria, p. 23.
[8] Gaylord, T. K. & Moharam, M. G. (1981). *Appl. Opt.*, *20*, 3271.
[9] Calvo, M. L. & Egido, P. J. D. (1982). *J. Mod. Opt.*, *29*, 1061.
[10] Born, M. & Wolf, E. (1980). *Principles of Optics*, 6th ed., Pergamon: Oxford.